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GAS-DYNAMIC ACCELERATION OF IONS IN AN INHOMOGENEOUS MAGNETIC FIELD

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1. Introduction. The basic features of gas-dynamic acceleration of ions in a homogeneous magnetic field were deduced in [1-4], wherein the existence of a Debye discontinuity was indicated, and self-similar solutions constructed, permitting description of ion acceleration for "steplike" application of the accelerating voltage. The form of the potential well for oscillating electrons found in [2] is shown in Fig. 1. The behavior of the potential at the Debye discontinuity is time-independent, while in the region $\phi < \phi_D$ the width of the well increases linearly with time.

As was shown in [3, 4], the efficiency of ion acceleration depends significantly on the thickness of the anode foil. Therefore it is desirable to select the foil such that the electrons transfer their energy to the accelerated ions significantly more rapidly than they lose energy to the foil. However reduction in foil thickness leads not only to increased ion acceleration efficiency, but also to a reduction in angular scattering of electrons within the foil, which finally leads to cutoff of the diode and a reduction in ion current density [4].

In order to weaken the corresponding diode cutoff limitation and increase the efficiency of energy transfer to ions [3] proposed a method of gas-dynamic acceleration of ions in an inhomogeneous magnetic field, of high level in the diode region, but weak in the acceleration region (Fig. 2). An electron beam with supercritical current is injected into the drift chamber between the sandwich of foils A and F, the space between which is filled by a neutralizing plasma. Under such conditions a large portion of the electrons injected into the chamber are reflected and begin to oscillate between the real cathode and a "virtual cathode" which appears in the drift chamber beyond foil F. As a result a dense cloud of oscillating electrons is formed near foils A and F. Under certain conditions the electrons may produce on the surface of foil F, located in the weak magnetic field region, a layer of plasma P, which serves as an ion source. Under the action of the electric field a cloud of ions is extracted from this plasma, and compensating the space charge of the oscillating electrons, is accelerated along the chamber.

The present study will evaluate the method of gas-dynamic acceleration of ions in an inhomogeneous magentic field proposed in [3] in two variants — a strongly scattering and nonscattering foil F. In the first variant the presence of the inhomogeneous magnetic field leads to an increase in ion current related to increase in the flux area in the acceleration region, while in the second the increase in current is insignificant, but nevertheless the efficiency of ion acceleration is increased, because all the energy of the oscillating electrons in the accelerated region will be included in a longitudinal degree of freedom. This fact leads to an increase in the rate of expansion of the plasma synthesized from ions and oscillating electrons. Because of this increase in the mean rate of plasma expansion the efficiency of acceleration increases also.

2. Oscillating Electron Distribution Function. We will consider ion acceleration for the case where the anode foil is strongly scattered and foil F is superthin. The thickness of the anode foil is then such that the following relationships are satisfied:

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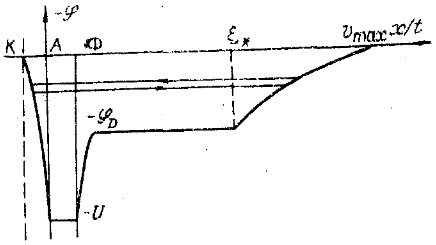

Fig. 1

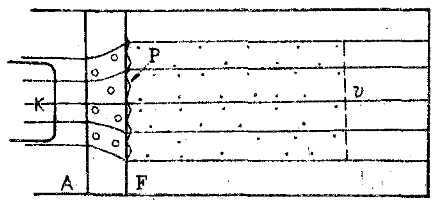

Fig. 2

$$\langle\theta^2\rangle \gg \delta W/W \gg d/c\tau; \quad (2.1)$$

$$\langle\theta^2\rangle \gg (\gamma m/M)^{1/2}. \quad (2.2)$$

Here $\gamma = 1 + W/mc^2$ is the relativistic beam factor; W is the energy of the original electron beam; $\langle\theta^2\rangle$, δW are the mean square of the scattering angle and the mean electron loss for normal incidence on the anode foil; τ is the duration of the accelerating impulse; d is the distance between the cathode and foil F; M is the mass of the ions. Condition (2.1) implies that electrons captured in the Debye layer are retarded over a time t, much less than the duration of the accelerating pulse Φ, as a result of which a steady-state electron distribution function is established [3, 4]. When condition (2.2) is satisfied the electron distribution function within the diode is isotropic $f(p, \theta) = f(p)$.

As for the foil F, its thickness is limited below by the condition

$$\delta W_1 \ll \delta W, \quad \langle\theta_1^2\rangle \ll \delta W_1/\delta W R, \quad (2.3)$$

where $\langle\theta_1^2\rangle$ and δW_1 are the mean square scattering angle and mean energy loss for normal incidence of an electron on foil F; $R = H_1/H_0$; H_1 is the field within the diode; H_0 is the field in the acceleration region. Condition (2.3) implies that the oscillating electron distribution function, while remaining isotropic within the diode, at $R \gg 1$ becomes almost one-dimensional on the foil F:

$$f(p, \theta_1) = \begin{cases} f(p), & \theta_1 < R^{-1/2} \\ 0, & \theta_1 > R^{-1/2} \end{cases}$$

(where θ_1 is the electron pitch angle on the foil F).

The state of the oscillating electron cloud can be characterized by the distribution function of these electrons on the anode foil f(p). The kinetic equation for f(p) can be found from the condition of conservation of the longitudinal adiabatic invariant

$$I(p, \theta, t) = \int_0^{x_1} q_\parallel (x, p, \theta)\, dx$$

with the additional condition $q_\perp = p \sin\theta/R^{1/2}$, which reflects the fact of constancy of the transverse component of the particle momentum. Here x_1 is the coordinate of the right rotation point, the lower limit is replaced by zero, since we are considering times for which $x_1 \gg d$; p and θ are the momentum and pitch angle of the electron on the anode foil; $q_\parallel (x, p, \theta)$ is the longitudinal momentum of the electron at the point x_1,

$$q_\parallel (x, p, \theta) = \left(q^2(x, p) - q_\perp^2\right)^{1/2};$$

q(x, p) is the total momentum of the electron at the point x,

$$q(x, p) = \left\{\left[(p^2 + m^2c^2)^{1/2} + \frac{e}{c}(\varphi - U)\right]^2 - m^2c^2\right\}^{1/2}.$$

We note that in a self-similar solution the potential φ depends on coordinate x and time t only in the combination $x/t = \xi$. Then for an electron with momentum $p > p_D$ the adiabatic invariant is a linear function of time $I(p, \theta, t) = tJ(p, \theta)$ [2], and the quantity

$$p_D = \left[2me(U - \varphi_D) + \frac{e^2}{c^2}(U - \varphi_D)^2\right]^{1/2}$$

is the limiting momentum at which an electron ejected along the normal to the foil surface is still retained within the Debye layer. For electrons with momentum $p < p_D$ $I(p, \theta, t) = 0$, since $x_1 = 0$.

In this case the kinetic equation for the distribution function has the form [4]

$$J_2(p)\frac{\partial f}{\partial p} + \frac{J_1(p)}{p} f + \frac{1}{p^2}\frac{\partial \delta W p^2 f}{\partial p} + Q = 0, \tag{2.4}$$

where

$$J_1(p) = J\left(p, \theta = \frac{\pi}{2}\right) = p\int_0^{\xi_1}\left(\frac{q^2}{p^2} - \frac{1}{R}\right)^{1/2} d\xi_1 \tag{2.5}$$

$$J_2(p) = \int_0^{\frac{\pi}{2}} J(p, \theta)\sin\theta\cos\theta d\theta.$$

The source Q describes the increase in number of particles within the cloud due to the beam:

$$Q = \frac{n_b v_0}{4\pi p^2}\delta(p - p_0).$$

Here n_b is the beam density at the anode foil; p_0 and v_0 are the momentum and velocity of the electron corresponding to an accelerating voltage U:

$$p_0 = \left(2meU + \frac{e^2U^2}{c^2}\right)^{1/2}, \quad v_0 = \frac{p_0 c}{(p_0^2 + m^2c^2)^{1/2}}.$$

Substituting $J(p, \theta)$ in Eq. (2.5) and integrating the latter over θ, we find

$$J_2(p) = \frac{pR}{3}\left[\int_0^{\xi_1}\frac{q^3}{p^3}d\xi - \int_0^{\xi_2}\left(\frac{q^2}{p^2} - \frac{1}{R}\right)^{\frac{3}{2}} d\xi\right],$$

where $\xi_1 = x_1/t$, and ξ_2 is defined by the equation $q(\xi_2, p) = pR^{-1/2}$. Solving Eq. (2.4), we obtain

$$f(p) = \frac{n_b v_0}{4\pi w(p_0)}\exp\left[\int_p^{p_0}\left(J_1 p + \frac{\partial \delta W p^2}{\partial p}\right)\frac{dp}{w(p)}\right] \tag{2.6}$$

$(w(p) = J_2(p)p^2 + \delta W p^2)$. We will call attention to the fact that at $p < p_D$ $I(p, \theta, t) = 0$ so that $f(p) = \text{const}/(p^2\delta W)$. The value of f(p) on the anode foil allows us to find the electron density at the point with potential ϕ:

$$n(\varphi, H) = 2\pi\int\frac{f(p(q))\, q dq d\mu H}{(q^2 - \mu H)^{1/2}} \quad (\mu = q_\perp^2/H). \tag{2.7}$$

We note that according to Eq. (2.6) the density of the oscillating electron cloud is proportional to the electron beam density on the anode foil,

$$n(\varphi, H) = n_b \nu(\varphi, H), \tag{2.8}$$

where the dimensionless function $\nu(\phi, H)$ is independent of n_b. Knowing the function $n(\phi, H)$, the self-similar solution of the equations describing gas-dynamic acceleration of the ions can be fully concretized.

3. Determination of Ion Flux Parameters. In one-dimensional formulation the problem of ion acceleration in self-similar variables is described by the system of equations [1]

$$(v - \xi)\frac{dn}{d\xi} + n\frac{dv}{d\xi} = 0, \quad (v - \xi)\frac{dv}{d\xi} + \frac{e}{M}\frac{d\varphi}{d\xi} = 0, \tag{3.1}$$

where v, n are the velocity and density of ions. From the condition of quasineutrality of the cloud the ion density is equal to the electron density:

$$n = n(\varphi, H_0). \tag{3.2}$$

System (3.1), (3.2) must be supplemented by boundary conditions [1]

$$v_* = \left[\frac{2(W - e\varphi_D)}{M}\right]^{1/2}, \qquad \int_{\varphi_D}^{U} n\,d\varphi = n_b v_0 \int_{\varphi_D}^{U} \frac{d\varphi}{\left[\frac{2(W - e\varphi_D)}{M}\right]^{1/2}}, \tag{3.3}$$

the first of which defines the electron velocity beyond the Debye discontinuity, while the second indicates the equality to zero of the electric field in the homogeneous flux region beyond the discontinuity.

For a specified beam density n_b and known function $\nu(\phi, H)$ system (3.1)-(3.3) completely defines the solution of the gas-dynamic portion of the problem. The major difficulties lie in self-consistent search for the function $\nu(\phi, H)$: it is determined by the electron distribution function, which in turn is determined by the dependence of the potential on ξ. In the majority of cases the corresponding probem can only be solved numerically by the successive approximation method: specification of the potential well form $\phi(\xi)$ in the zeroth step, use of Eqs. (2.6), (2.8) to calculate the distribution function f(p) and the electron density $n_b\nu(\phi, H)$. Knowing the function $\nu(\phi, H)$, from system (3.1)-(3.3) we obtain the following approximation for $\phi(\xi)$. Repeating this procedure several times, we can construct a sufficiently accurate expression for f(p) and ν. As a result, a self-consistent solution of the problem is obtained, defined to the accuracy of a scale factor n_b, which is found by solution of the Poisson equation within the diode.

In addition there is a case in which the solution can be determined analytically, this being the case of nonrelativistic electrons, $\gamma - 1 \ll 1$. We will now consider the situation in which $R \gg 1$, and the electron energy losses in the anode foil are significantly less than the loss of electron energy expended in ion acceleration:

$$\frac{\delta W}{W} \ll \left(\frac{\gamma m}{M}\right)^{1/2}. \tag{3.4}$$

At $R \gg 1$ $J_1(p) = 2J_2(p) = J(p, \theta = 0)$. In the nonrelativistic case $p^2\delta W$ is independent of electron energy [4], so that the distribution function has the form

$$f(p) = \begin{cases} \dfrac{n_b v_0}{2\pi J_0 p_D^2}, & 0 < p < p_D, \\ \dfrac{n_b v_0}{2\pi J_0 p^2}, & p_D < p < p_0 \end{cases} \tag{3.5}$$

($J_0 = J(p_0, \theta = 0)$). Substituting the distribution function obtained into Eq. (2.7), we calculate the density of the oscillating electron cloud in the acceleration region

$$n(\psi, H_0) = \begin{cases} n_0\psi^{1/2}, & \psi < \psi_D, \\ n_0\left[\psi^{1/2} - \dfrac{2(\psi - \psi_D)^{3/2}}{3(1 - \psi_D)}\right], & \psi > \psi_D, \end{cases} \tag{3.6}$$

where $\psi = \phi/U$ is the dimensionless potential; $n_0 = 2n_bW/J_0R$. The quantity n_0 can be determined by use of the fact that in view of the quasineutrality of the flux of the beam current density en_bv_0 is equal to en_*v_*R, the density of the ion current, where n_* and v_* are the ion density and velocity at $\psi = \psi_D$. Thus we arrive at the result

$$n_0 = \frac{n_b}{R\left[\frac{m}{M}\psi_D(1 - \psi_D)\right]^{1/2}}.$$

We determine the potential discontinuity beyond the Debye layer from boundary conditions (3.3):

$$5(1-\psi_D^{3/2})-2(1-\psi_D)^{3/2}=15\psi_D^{1/2}(1-\psi_D). \quad (3.7)$$

Substituting the expression $n(\psi, H_0)$ in system (3.1), we find the self-similar solution in the region of the rarefaction wave $v_{max} > \xi > \xi_*$ (see Fig. 1 for notation):

$$n=n_0(v_{max}-\xi)/2v_0,\quad v=(v_{max}+\xi)/2,$$
$$\psi=(v_{max}-\xi)^2/4v_0^2. \quad (3.8)$$

Here $v_0=\left(\frac{2W}{m}\right)^{1/2}$; $v_{max}=v_0\left[(1-\psi_D)^{1/2}+\psi_D^{1/2}\right]$; $\xi_*=v_0\left[(1-\psi_D)^{1/2}-\psi_D^{1/2}\right]$. The values of n_* and v_* in the constant flow region are determined by merging this flow with the rarefaction wave:

$$v_*=(v_{max}+\xi_*)/2,\quad n_*=n_0(v_{max}-\xi_*)/2v_0. \quad (3.9)$$

Solving the alegrbaic system (3.7)-(3.9), we find all numerical parameters not yet defined:

$$\psi_D=0{,}047,\ \xi_*=0{,}755,\ v_{max}=1{,}19,\ v_*=0{,}976,\ n_*=0{,}217. \quad (3.10)$$

Thus the problem of gas-dynamic acceleration of ions has been completely solved, and to determine the beam density n_b we solve the Poisson equation within the diode

$$\frac{d^2\psi}{dx^2}=\frac{4\pi e}{U}\left[\frac{n_b}{\psi^{1/2}}+n(\psi, H_1)\right],\quad \psi|_{x=0}=0,\quad \left.\frac{d\psi}{dx}\right|_{x=0}=0,\quad \psi|_{x=d_1}=1, \quad (3.11)$$

where the first term on the right considers the electron beam charge density and the second considers the oscillating electron charge density. The electron cloud density in the diode

$$n(\psi, H_1)=\begin{cases} n_1\left[\psi^{1/2}-(1-\psi)^{1/2}\operatorname{arctg}\left(\frac{\psi}{1-\psi}\right)^{1/2}\right], & \psi<\psi_D,\\ n_1\left\{\psi^{1/2}-(\psi-\psi_D)^{1/2}-(1-\psi)^{1/2}\left[\operatorname{arctg}\left(\frac{\psi}{1-\psi}\right)^{1/2}-\right.\right. & \\ \left.\left.-\operatorname{arctg}\left(\frac{\psi-\psi_D}{1-\psi}\right)^{1/2}\right]+\frac{(\psi-\psi_D)^{3/2}}{3(1-\psi_D)}\right\}, & \psi>\psi_D \end{cases} \quad (3.12)$$

($n_1 = 2n_0R$). Substituting $n(\psi, H_1)$ in Eq. (3.11) and integrating over ψ, we obtain $n_b = 0.256n_{b0}$, where n_{b0} is the electron beam density determined by the "3/2" law.

An important characteristic of the acceleration process is the fraction of energy transferred to ions by the beam electrons:

$$\eta=\frac{\frac{M}{2}\int_0^{x_1} nv^2dx}{n_bWv_0t}. \quad (3.13)$$

In the case considered the efficiency of acceleration $\eta = 0.93$. We note that for a nonrelativistic diode in a homogeneous magnetic field $\eta = 0.77$, while $n_b = 0.234n_{b0}$ [3, 4]. The electron energy loss in the foil is then significantly greater than the energy conveyed to accelerated ions by the beam electrons:

$$\frac{\delta W}{W}\gg\left(\frac{\gamma m}{M}\right)^{1/2}, \quad (3.14)$$

for R >> 1 the problem of gas-dynamic acceleration of ions can also be solved analytically. In this case the efficiency will be

$$\eta=0{,}195\frac{W}{\delta W}\left(\frac{m}{M}\right)^{1/2}.$$

We note that this value is significantly greater than the efficiency of ion acceleration in a homogeneous magnetic field:

$$\eta = 0{,}119 \frac{W}{\delta W} \left(\frac{m}{M}\right)^{1/2}.$$

The beam density n_b for a diode located in an inhomogeneous magnetic field will be the same as that of a diode in a homogeneous magnetic field when condition (3.14) is satisfied.

With an arbitrary relationship between $\delta W/W$ and $(\gamma m/M)^{1/2}$ for $R \gtrsim 1$ the solution of the problem can only be constructed numerically by the method of successive approximations. We will present the results of numerical calculations performed for conditions (2.1)-(2.3). As was noted above, the distribution function, while remaining isotropic in the diode, becomes almost one-dimensional in the acceleration region for $R >> 1$, with the major fraction of the oscillating electron energy is contained in the longitudinal degree of freedom. This fact leads to an increase in the mean rate of expansion of the ion flux, and consequently, an increase in the efficiency of ion acceleration.

The dependence of ion acceleration efficiency η on compression of the magnetic field R at $\gamma = 3$, $\delta W/mc^2 = 4 \cdot 10^{-3}$ is shown in Fig. 3. In this case the increase in diode current with increase in R is insignificant; thus, at R = 5 the increase in current as compared to R = 1 comprises about 10%.

We will note that the results obtained in this section are valid for

$$R \ll (M/\gamma m)^{1/2}. \tag{3.15}$$

The limitation on R is related to the following fact. Because of expansion of the potential well electrons with a momentum $p > p_D$, leaving the anode foil at an angle $\theta \not\approx \pi/2$ lose a portion of their longitudinal momentum and no longer return to the diode. Since in deriving the kinetic equation these electrons were not considered, it is necessary that they be low in number. It can easily be seen that this will be the case when inequality (3.15) is satisfied.

4. Ion Acceleration in a Diode with Strongly Scattering Foil F. We will present results of a calculation of gas-dynamic acceleration of ions for the case where the anode foil is superthin and the foil F is strong scattering:

$$\langle\theta_1^2\rangle \gg \frac{\delta W_i}{W} \gg \frac{d}{c\tau}, \tag{4.1}$$

$$\langle\theta_1^2\rangle \gg \max\left(\left(\frac{\gamma m}{M}\right)^{1/2}, R^{-1}\right), \quad \delta W \ll \delta W_i.$$

Condition (4.1) indicates that elastic scattering of electrons in foil F is a very rapid process, so that the oscillating electron distribution function is isotropic in both the diode and the acceleration region. Therefore all results involving the form of the spectrum and the level of ion acceleration efficiency remain the same as in [4], which considered ion acceleration in a homogeneous magnetic field.

However the presence of an inhomogeneous magnetic field leads to a significant reduction in the density of the electron cloud within the diode, related to increase in the area of the flux in the ion acceleration region. Decrease in the oscillating electron cloud density in turn leads to an increase in the beam current j_b, and finally, to an increase in ion current.

For a nonrelativistic diode $\gamma - 1 << 1$, the dependence of diode current on R was found in [3]:

$$\frac{j_b}{j_{b0}} = 14{,}5 s^{3/4} (1 - 0{,}8 s^{1/8})^2.$$

Here $s = R\left[1{,}17\left(\frac{m}{M}\right)^{1/2} + 7{,}5\frac{\delta W_i}{W}\right]$; ; j_{b0} is the diode current density defined by the "3/2" law.

For a relativistic diode the solution can only be obtained by numerical calculations using the method presented in Section 3. The dependence of j_b/j_{b0} on R at $\gamma = 3$, $\delta W_i/mc^2 = 4 \cdot 10^{-3}$ is shown in Fig. 4, where j_{b0} is the diode current determined in analogy to the "3/2" law for a relativistic diode. As is evident from Fig. 4, the presence of an inhomogeneous magnetic field leads to a significant increase in diode current density.

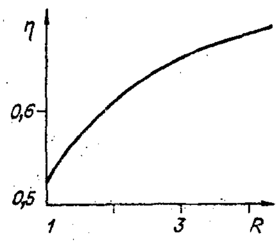

Fig. 3

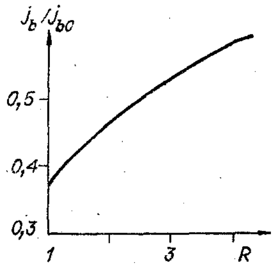

Fig. 4

The results obtained above are in good agreement with the experimental results presented in [5]. It should be understood that under real conditions one does not deal with a strictly "step" voltage form, so that comparison of the presult results with experimental data can only be qualitative in character. Another possible limitation on the applicability of the proposed treatment is that in the present study the possibility of reduction of the diode impedance by neutralization of the diode gap electron charge by the charge of ions emitted from the inner surface of the ion foil was not considered.

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